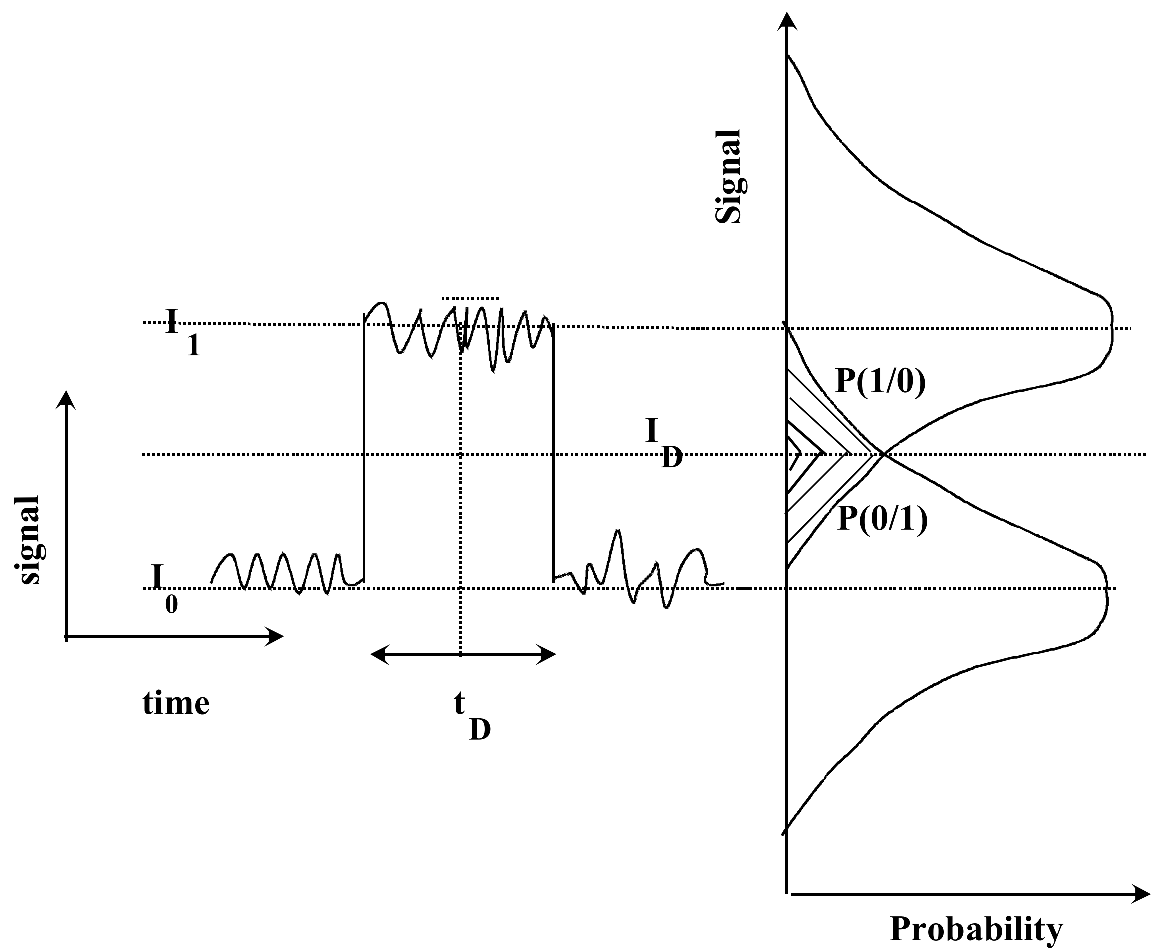
RECEIVER SENSITIVITY:

A measure of a ***good receiver*** is to have the *same performance* with the *lowest level of incident optical power.*

- ***Performance*** can be measured as a *low bit error rate* (BER).
- ***BER ≡ probability of an incorrect identification of a bit by the decision circuit of a receiver.***
- A BER of 3×10^{-7} implies that 3 bits will be in error for every 10 million received bits.
- For communications systems a criteria used is that the BER $\leq 10^{-9}$.

Receiver Sensitivity ≡ *minimum average received optical power* $\overline{P}_{\min}$ required to achieve a fixed BER.



- If $I > I_D$ a 1 bit is recorded.
- If $I < I_D$ a 0 bit is registered.
- An error occurs for a 1 bit if $I < I_D$ and similarly an error for a zero bit takes place when $I > I_D$.

These errors can be included in the error probability as

$$BER = p(1)P(0/1) + p(0)P(1/0)$$

- p(1) and p(0) are the probabilities of receiving a 1 and 0 bit respectively
- P(0/1) is the probability of deciding 0 when a 1 is received and P(1/0) is the probability of deciding 1 when a 0 is received.
- Assuming that 1s and 0s are equally likely, p(1) = p(0) = 0.5

$$BER = 0.5[P(0/1) + P(1/0)].$$

Both *shot* and *thermal noise* can be modeled with *Gaussian statistics* (for large numbers of electrons).

- The total *variance in the current* for I_1 or I_0 can be modeled as

$$\sigma_{1,0}^2 = \langle i_{s,1,0}^2 \rangle + \langle i_{T,1,0}^2 \rangle$$

- The *average noise currents* squared were previously described for shot and thermal noise.
- Since the *average current I_p* is different for the 1 and 0 levels the shot noise and the variance will be different.
- Consequently in general σ_1 and σ_0 will also differ.

- The ***conditional probabilities*** based on Gaussian statistics are:

$$P(0/1) = \frac{1}{\sigma_1\sqrt{2\pi}} \int_{-\infty}^{I_D} \exp\left(-\frac{(I - I_1)^2}{2\sigma_1^2} \right) dI = \frac{1}{2} erfc\left(\frac{I_1 - I_D}{\sigma_1\sqrt{2}} \right)$$

$$P(1/0) = \frac{1}{\sigma_0\sqrt{2\pi}} \int_{I_D}^{\infty} \exp\left(-\frac{(I - I_0)^2}{2\sigma_0^2} \right) dI = \frac{1}{2} erfc\left(\frac{I_D - I_0}{\sigma_0\sqrt{2}} \right)$$

$$erfc(y) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} \exp(-y)^2 dy$$

This results in a BER of:

$$BER = \frac{1}{4}\left[erfc\left(\frac{I_1 - I_D}{\sigma_1\sqrt{2}} \right) + erfc\left(\frac{I_D - I_0}{\sigma_0\sqrt{2}} \right) \right].$$

I_D is chosen to minimize BER. This occurs when

$$(I_1 - I_D)/\sigma_1 = (I_D - I_0)/\sigma_0 \equiv Q$$

and

$$I_D = \frac{\sigma_0 I_1 + \sigma_1 I_0}{\sigma_0 + \sigma_1}.$$

- When $\sigma_1 = \sigma_0$, $\rightarrow$ $I_D = (I_1 + I_0)/2$

or the *decision threshold current* I_D lies between I_0 and I_1.

In this case:

$$BER = \frac{1}{2} erfc\left(\frac{Q}{\sqrt{2}} \right) \approx \frac{\exp\left(-Q^2/2 \right)}{Q\sqrt{2\pi}}.$$

The approximation is good for Q > 3.

Solving for Q:

$$Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0}$$

- A BER $\approx 10^{-9}$ occurs with Q ~ 6.

<u>Minimum Received Optical Power:</u>

Since the currents I_1 and I_0 are related to the optical power through the responsivity, it is possible to determine the *minimum optical power* to obtain a *fixed BER*.

For the case with $I_0 = 0$, $P_0 = 0$. The detector current for the 1 bit is

$$I_1 = MRP_1 = 2MR\overline{P}_{\min},$$

where P_{min} is the *average received minimum power* with

$$\overline{P}_{\min} = (P_1 + P_0)/2.$$

Noise terms include both *thermal* and *shot noise* for the 1 bit *but only thermal noise* for the *0 bit*:

$$\sigma_1 = \left(\sigma_s^2 + \sigma_T^2\right)^{1/2}; \sigma_0 = \sigma_T \,.$$

Therefore:

$$Q = \frac{I_1}{\sigma_1 + \sigma_0} = \frac{2MR\overline{P}_{\min}}{\left(\langle i_s^2 \rangle + \langle i_T^2 \rangle\right)^{1/2} + < i_T^2 >^{1/2}} \,.$$

Solving for P_{min}

$$P_{\min} = \frac{Q}{R}\left(qF_A Q B_e + \frac{\langle i_T^2 \rangle^{1/2}}{M} \right)$$

<u>Quantum Limit for Photodetection</u>:

- An <u>ideal photodiode</u> *(no thermal noise, I_D, and 100% QE)*, has $\sigma_0 = 0$ (*no shot noise*).
- In this case $I_0 \sim 0$, very few photons (***even 1!***) can be detected provided that they produce *e-h pairs*.
- $\therefore$ In this case *Gaussian statistics* are no longer valid.
- *Poisson statistics* should be applied.

Assuming that N_p is the *average number of photons* in each 1 bit (0 are in a 0 bit) the probability for the formation of *m, e-h hole pairs* is given by *Poisson distribution*

$$P_m = \exp(-N_p) N_p^m / m!$$

- Using our earlier definitions, P(1/0) = 0 since no *e-h pairs* are formed when $N_p = 0$.

- $P(0/1) = \exp(-N_p)$

In this case: *m* = 0 since a 0 is decided in this case even though a 1 is received.

$$\therefore BER = \exp(-N_p)/2 .$$

The ***minimum received optical power*** in this case is

$$P_{\min} = N_p h\nu B/2 .$$

- At 1.55 μm hν = 0.8 eV, and the P_{min} = 13 nW (-48.9 dBm @ 10 Gb/s).
- In most receivers however N_p is typically ~ 1000 which is considerably higher than this limit.

Extinction Ratio:
In many cases the power in the 0-bit will not be 0.

- *Extinction ratio* – can be defined as

$$r = P_o / P_1 .$$

- In this case

$$Q = \left(\frac{1-r}{1+r}\right)\frac{2RP_{\min}}{\sigma_1 + \sigma_0}$$

- *Minimum required optical power* becomes

$$P_{\min} = \left(\frac{1+r}{1-r}\right)\cdot\frac{(\sigma_1 + \sigma_0)}{2}\cdot\left(\frac{Q}{R}\right)$$

Effect of Intensity Noise:

- Most optical sources will exhibit some form of *output power fluctuation*.
- This fluctuation is another source of system noise and must be considered in the SNR.
- Assuming that all noise sources are independent the *total current noise power* term can be represented as:

$$\langle i_N^2 \rangle = \langle i_s^2 \rangle + \langle i_T^2 \rangle + \langle i_I^2 \rangle .$$

- The *laser power fluctuation* can be expressed as:

$$\langle i_I \rangle = R \langle (\Delta P_{inc}^2) \rangle^{1/2} = R P_{inc} r_I ,$$

with

$$r_I^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} RIN(\omega) d\omega .$$

RIN(ω) was described previously.

- $r_I = 1/SNR$ for light emitted from the laser source.
- A source with SNR>20 dB has $r_I < 0.01$.

- With ***source noise*** the ***Q parameter*** becomes (with $I_0 = 0$ and $\sigma_0 = \sigma_T$):

$$Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0} = \frac{2R\overline{P}_{rec}}{\left(\left\langle\sigma_T^2\right\rangle + \left\langle\sigma_s^2\right\rangle + \left\langle\sigma_I^2\right\rangle\right)^{1/2} + \sigma_T}$$

with

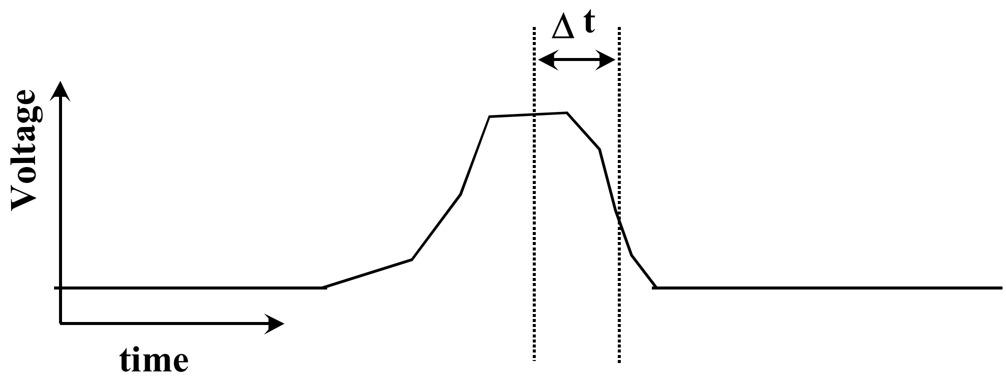
$$\sigma_s = \left(4qR\overline{P}_{rec}B_e\right)^{.5}; \sigma_I = 2r_I R\overline{P}_{rec}$$

- The ***receiver sensitivity*** becomes:

$$\overline{P}_{rec}\left(r_I\right) = \frac{Q\sigma_T + Q^2 qB_e}{R\left(1 - r_I^2 Q^2\right)}$$

Effects of Timing Jitter:

- The previous receiver sensitivity analysis is based on the assumption that the *signal is sampled at the peak of the voltage pulse*.
- Timing decisions are typically determined by a *clock-recovery circuit*.
- Since the input to this circuit is noisy the *sampling time fluctuates* from bit to bit.
- This fluctuation is referred to as ***timing jitter*** and will degrade the SNR.



- Since Δt is a *random variable* the *change in the sampled voltage level* is also a *random variable*.

- This reduces the receiver performance.

- The SNR can be improved at the expense of higher source power.

Effect on System Performance:

- Assume a *pin* detector/receiver that is limited by thermal noise with (extinction ratio) r = 0, and $I_0 = 0$.

$$Q = \frac{I_1 - \langle \Delta i_j \rangle}{\left(\sigma_T^2 + \sigma_j^2\right)^{.5} + \sigma_T}$$

- assuming that the BΔt << 1 where B = bit rate = $1/T_B$,

$$\Delta i_j = \left(2\pi^2/3 - 4\right)\cdot\left(B\Delta t\right)^2 I_1 .$$

Defining

$$b = \left(4\pi^2/3 - 8\right)\left(B\tau_j\right)^2$$

with τ_j the RMS value of Δt, the receiver sensitivity becomes

$$\overline{P}_{rec}(b) = \left(\frac{\sigma_T Q}{R}\right)\frac{1-b/2}{(1-b/2)^2 - b^2Q^2/2}.$$

- A <u>*power penalty*</u> can be defined as the *increase in received power required to maintain the same BER as obtained without a timing jitter factor*.

$$\delta_j = 10\log_{10}\left(\frac{\overline{P}_{rec}(b)}{\overline{P}_{rec}(0)}\right) = 10\log_{10}\left(\frac{1-b/2}{(1-b/2)^2 - b^2Q^2/2}\right).$$

The *power penalty* is negligible when *timing jitter* $\tau_j < (0.1)T_B$ but increases rapidly beyond this point.