Electronics Tutorial about Passive Low Pass Filters

Passive Low Pass Filter

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Low Pass Filter Introduction

Basically, an electrical filter is a circuit that can be designed to modify, reshape or reject all unwanted frequencies of an electrical signal and accept or pass only those signals wanted by the circuits designer. In other words they "filter-out" unwanted signals and an ideal filter will separate and pass sinusoidal input signals based upon their frequency.

In low frequency applications (up to 100kHz), passive filters are generally constructed using simple RC (Resistor-Capacitor) networks, while higher frequency filters (above 100kHz) are usually made from RLC (Resistor-Inductor-Capacitor) components. Passive filters are made up of passive components such as resistors, capacitors and inductors and have no amplifying elements (transistors, op-amps, etc) so have no signal gain, therefore their output level is always less than the input.

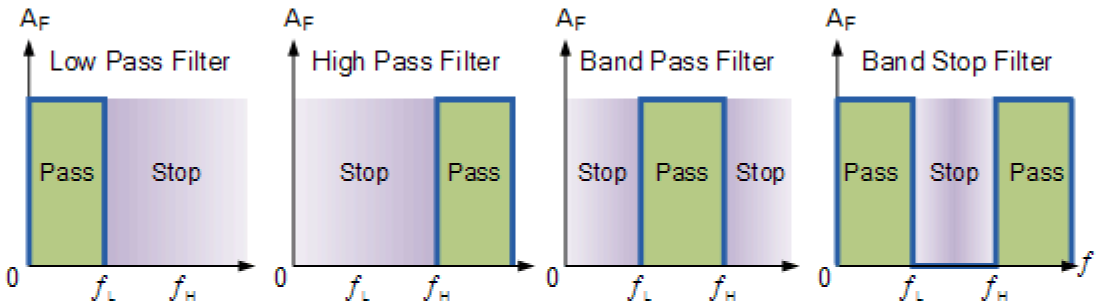
Filters are so named according to the frequency range of signals that they allow to pass through them, while blocking or "attenuating" the rest. The most commonly used filter designs are the:

- 1. The Low Pass Filter – the low pass filter only allows low frequency signals from 0Hz to its cut-off frequency, ƒc point to pass while blocking those any higher.
- 2. The High Pass Filter – the high pass filter only allows high frequency signals from its cut-off frequency, ƒc point and higher to infinity to pass through while blocking those any lower.
- 3. The Band Pass Filter – the band pass filter allows signals falling within a certain frequency band setup between two points to pass through while blocking both the lower and higher frequencies either side of this frequency band.

Simple First-order passive filters (1st order) can be made by connecting together a single resistor and a single capacitor in series across an input signal, (Vin) with the output of the filter, (Vout) taken from the junction of these two components. Depending on which way around we connect the resistor and the capacitor with regards to the output signal determines the type of filter construction resulting in either a **Low Pass Filter** or a **High Pass Filter**.

As the function of any filter is to allow signals of a given band of frequencies to pass unaltered while attenuating or weakening all others that are not wanted, we can define the amplitude response characteristics of an ideal filter by using an ideal frequency response curve of the four basic filter types as shown.

Ideal Filter Response Curves



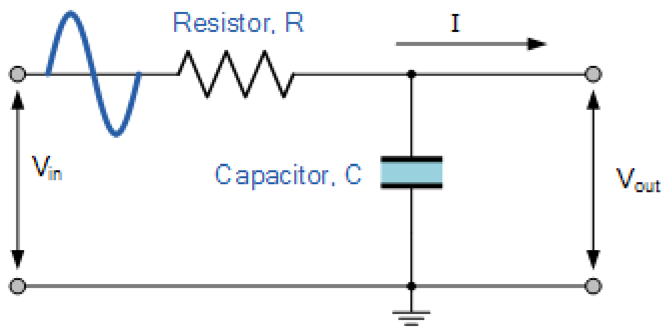
Filters can be divided into two distinct types: active filters and passive filters. Active filters contain amplifying devices to increase signal strength while passive do not contain amplifying devices to strengthen the signal. As there are two passive components within a passive filter design the output signal has a smaller amplitude than its corresponding input signal, therefore passive RC filters attenuate the signal and have a gain of less than one, (unity).

A Low Pass Filter can be a combination of capacitance, inductance or resistance intended to produce high attenuation above a specified frequency and little or no attenuation below that frequency. The frequency at which the transition occurs is called the "cutoff" frequency. The simplest low pass filters consist of a resistor and capacitor but more sophisticated low pass filters have a combination of series inductors and parallel capacitors. In this tutorial we will look at the simplest type, a passive two component RC low pass filter.

The Low Pass Filter

A simple passive **RC Low Pass Filter** or **LPF**, can be easily made by connecting together in series a single Resistor with a single Capacitor as shown below. In this type of filter arrangement the input signal (Vin) is applied to the series combination (both the Resistor and Capacitor together) but the output signal (Vout) is taken across the capacitor only. This type of filter is known generally as a "first-order filter" or "one-pole filter", why first-order or single-pole?, because it has only "one" reactive component, the capacitor, in the circuit.

RC Low Pass Filter Circuit



As mentioned previously in the *Capacitive Reactance* tutorial, the reactance of a capacitor varies inversely with frequency, while the value of the resistor remains constant as the frequency changes. At low frequencies the capacitive reactance, (Xc) of the capacitor will be very large compared to the resistive value of the resistor, R and as a result the voltage across the capacitor, Vc will also be large while the voltage drop across the resistor, Vr will be much lower. At high frequencies the reverse is true with Vc being small and Vr being large.

While the circuit above is that of an RC Low Pass Filter circuit, it can also be classed as a frequency variable potential divider circuit similar to the one we looked at in the *Resistors* tutorial. In that tutorial we used the following equation to calculate the output voltage for two single resistors connected in series.

$$V_{out} = V_{in} \times \frac{R_2}{R_1 + R_2}$$

where: $R_1 + R_2 = R_T$, the total resistance of the circuit

We also know that the capacitive reactance of a capacitor in an AC circuit is given as:

$$X_C = \frac{1}{2\pi f C} \text{ in Ohm's}$$

Opposition to current flow in an AC circuit is called **impedance**, symbol Z and for a series circuit consisting of a single resistor in series with a single capacitor, the circuit impedance is calculated as:

$$Z = \sqrt{R^2 + X_C^2}$$

Then by substituting our equation for impedance above into the resistive potential divider equation gives us:

$$V_{out} = V_{in} \times \frac{X_C}{\sqrt{R^2 + X_C^2}} = V_{in} \frac{X_C}{Z}$$

So, by using the potential divider equation of two resistors in series and substituting for impedance we can calculate the output voltage of an RC Filter for any given frequency.

Example No1

A **Low Pass Filter** circuit consisting of a resistor of 4k7Ω in series with a capacitor of 47nF is connected across a 10v sinusoidal supply. Calculate the output voltage (Vout) at a frequency of 100Hz and again at frequency of 10,000Hz or 10kHz.

At a frequency of 100Hz.

$$Xc = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 100 \times 47 \times 10^{-9}} = 33{,}863\Omega$$

$$V_{OUT} = V_{IN} \times \frac{Xc}{\sqrt{R^2 + X_C^2}} = 10 \times \frac{33863}{\sqrt{4700^2 + 33863^2}} = 9.9v$$

At a frequency of 10kHz.

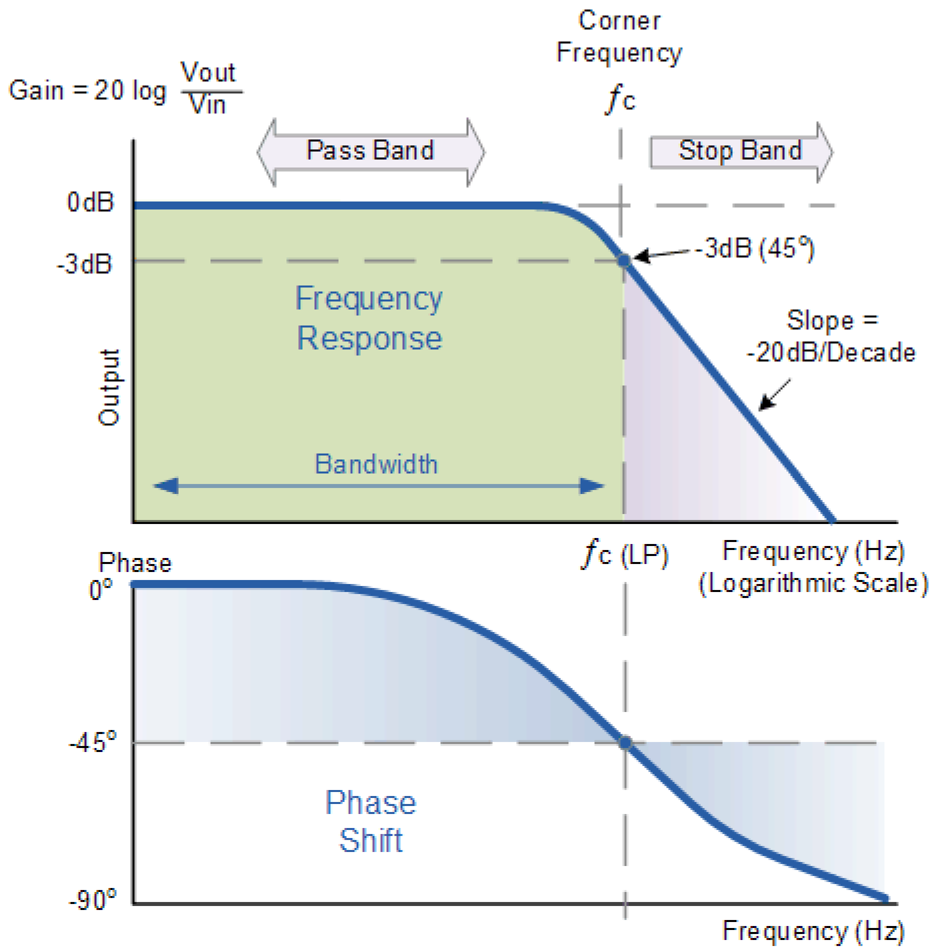
$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 10{,}000 \times 47 \times 10^{-9}} = 338.6\Omega$$

$$V_{OUT} = V_{IN} \times \frac{X_C}{\sqrt{R^2 + X_C^2}} = 10 \times \frac{338.6}{\sqrt{4700^2 + 338.6^2}} = 0.718v$$

Frequency Response

We can see above, that as the frequency increases from 100Hz to 10kHz, the output voltage (Vout) decreases from 9.9v to 0.718v. By plotting the output voltage against the input frequency, the **Frequency Response Curve** or **Bode Plot** function of the low pass filter can be found, as shown below.

Frequency Response of a 1st-order Low Pass Filter



The Bode Plot shows the **Frequency Response** of the filter to be nearly flat for low frequencies and all of the input signal is passed directly to the output, resulting in a gain of nearly 1, called unity, until it reaches its **Cut-off Frequency** point (ƒc). This is because the reactance of the capacitor is high at low frequencies and blocks any current flow through the capacitor.

After this cut-off frequency point the response of the circuit decreases giving a slope of -20dB/ Decade or (-6dB/Octave) "roll-off" as signals above this frequency become greatly attenuated, until at very high frequencies the reactance of the capacitor becomes so low that it gives the effect of a short circuit condition on the output terminals resulting in zero output.

For this type of **Low Pass Filter** circuit, all the frequencies below this cut-off, fc point that are unaltered with little or no attenuation and are said to be in the filters **Pass band** zone. This pass band zone also represents the **Bandwidth** of the filter. Any signal frequencies above this point cut-off point are generally said to be in the filters **Stop band** zone and they will be greatly attenuated.

This "Cut-off", "Corner" or "Breakpoint" frequency is defined as being the frequency point where the capacitive reactance and resistance are equal, R = Xc = 4k7Ω. When this occurs the output signal is attenuated to 70.7% of the input signal value or **-3dB** (20 log (Vout/Vin)) of the input. Although R = Xc, the output is **not** half of the input signal. This is because it is equal to the vector sum of the two and is therefore 0.707 of the input.

As the filter contains a capacitor, the Phase Angle (Φ) of the output signal **LAGS** behind that of the input and at the -3dB cut-off frequency (fc) and is -45^o out of phase. This is due to the time taken to charge the plates of the capacitor as the input voltage changes, resulting in the output voltage (the voltage across the capacitor) "lagging" behind that of the input signal. The higher the input frequency applied to the filter the more the capacitor lags and the circuit becomes more and more "out of phase".

The cut-off frequency point and phase shift angle can be found by using the following equation:

Cut-off Frequency and Phase Shift

$$f\text{c} = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 4700 \times 47 \times 10^{-9}} = 720\text{Hz}$$

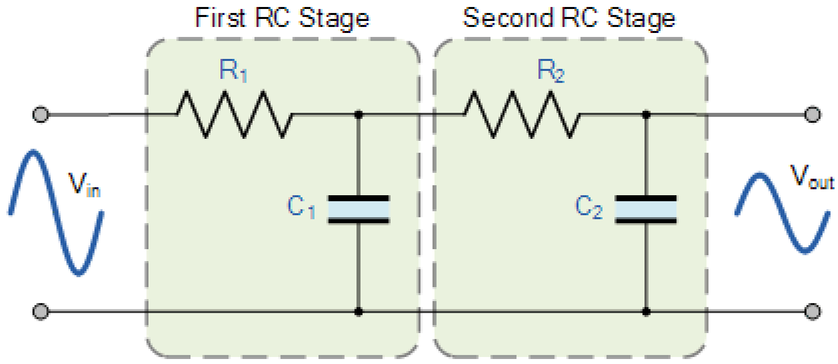
$$\text{Phase Shift } \varphi = \text{-arctan}\,(2\pi f RC)$$

Then for our simple example of a "**Low Pass Filter**" circuit above, the cut-off frequency (fc) is given as 720Hz with an output voltage of 70.7% of the input voltage value and a phase shift angle of -45^o.

Second-order Low Pass Filter

Thus far we have seen that simple first-order RC low pass filters can be made by connecting a single resistor in series with a single capacitor. This single-pole arrangement gives us a roll-off slope of -20dB/decade attenuation of frequencies above the cut-off point at f_{3dB} . However, sometimes in filter circuits this -20dB/decade (-6dB/octave) angle of the slope may not be enough to remove an unwanted signal then two stages of filtering can be used as shown.

Second-order Low Pass Filter



The above circuit uses two passive first-order low pass filters connected or "cascaded" together to form a second-order or two-pole filter network. Therefore we can see that a first-order low pass filter can be converted into a second-order type by simply adding an additional RC network to it and the more RC stages we add the higher becomes the order of the filter. If a number (n) of such RC stages are cascaded together, the resulting RC filter circuit would be known as an "n^{th}-order" filter with a roll-off slope of "n x -20dB/decade".

So for example, a second-order filter would have a slope of -40dB/decade (-12dB/octave), a fourth-order filter would have a slope of -80dB/decade (-24dB/octave) and so on. This means that, as the order of the filter is increased, the roll-off slope becomes steeper and the actual stop band response of the filter approaches its ideal stop band characteristics.

Second-order filters are important and widely used in filter designs because when combined with first-order filters any higher-order n^{th}-value filters can be designed using them. For example, a third order low-pass filter is formed by connecting in series or cascading together a first and a second-order low pass filter.

But there is a downside too cascading together RC filter stages. Although there is no limit to the order of the filter that can be formed, as the order increases, the gain and accuracy of the final filter declines. When identical RC filter stages are cascaded together, the output gain at the required cut-off frequency (*ƒ*c) is reduced (attenuated) by an amount in relation to the number of filter stages used as the roll-off slope increases. We can define the amount of attenuation at the selected cut-off frequency using the following formula.

Passive Low Pass Filter Gain at *ƒ*c

$$\left(\frac{1}{\sqrt{2}}\right)^n$$

where "n" is the number of filter stages.

http://www.electronics-tutorials.ws/filter/filter_1.html

So for a second-order passive low pass filter the gain at the corner frequency fc will be equal to 0.7071 x 0.7071 = 0.5Vin (-6dB), a third-order passive low pass filter will be equal to 0.353Vin (-9dB), fourth-order will be 0.25Vin (-12dB) and so on. The corner frequency, fc for a second-order passive low pass filter is determined by the resistor/capacitor (RC) combination and is given as.

2nd-Order Filter Corner Frequency

$$f_C = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}} \text{ Hz}$$

In reality as the filter stage and therefore its roll-off slope increases, the low pass filters -3dB corner frequency point and therefore its pass band frequency changes from its original calculated value above by an amount determined by the following equation.

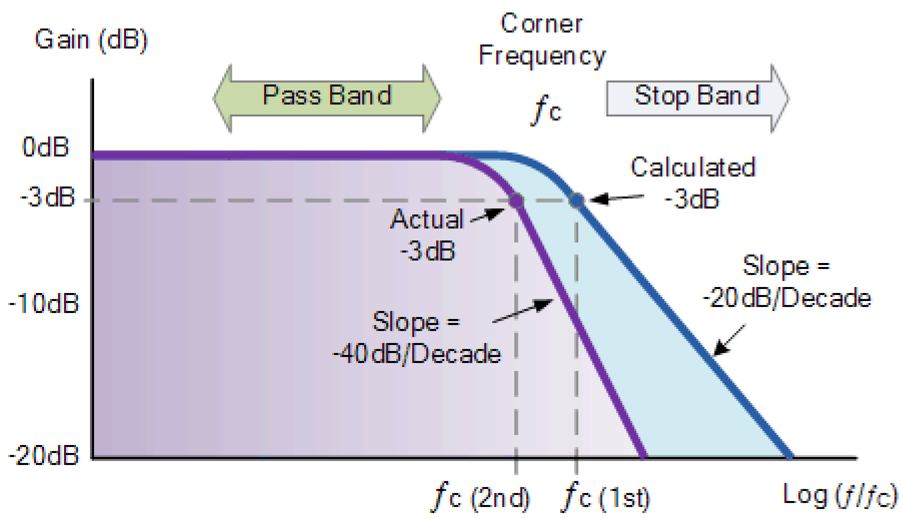
2nd-Order Low Pass Filter -3dB Frequency

$$f_{(-3dB)} = f_C\sqrt{2^{\left(\frac{1}{n}\right)} - 1}$$

where fc is the calculated cut-off frequency, n is the filter order and f_{-3dB} is the new -3dB pass band frequency as a result in the increase of the filters order.

Then the frequency response (bode plot) for a second-order low pass filter assuming the same -3dB cut-off point would look like:

Frequency Response of a 2nd-order Low Pass Filter



http://www.electronics-tutorials.ws/filter/filter_1.html

In practice, cascading passive filters together to produce larger-order filters is difficult to implement accurately as the dynamic impedance of each filter order affects its neighbouring network. However, to reduce the loading effect we can make the impedance of each following stage 10x the previous stage, so R2 = 10 x R1 and C2 = 1/10th C1. Second-order and above filter networks are generally used in the feedback circuits of op-amps, making what are commonly known as *Active Filters* or as a phase-shift network in *RC Oscillator* circuits.

Low Pass Filter Summary

So to summarize, the **Low Pass Filter** has a constant output voltage from D.C. (0Hz), up to a specified Cut-off frequency, (fc) point. This cut-off frequency point is 0.707 or **-3dB** (dB = -20log Vout/Vin) of the voltage gain allowed to pass. The frequency range "below" this cut-off point fc is generally known as the **Pass Band** as the input signal is allowed to pass through the filter. The frequency range "above" this cut-off point is generally known as the **Stop Band** as the input signal is blocked or stopped from passing through.

A simple 1st order low pass filter can be made using a single resistor in series with a single non-polarized capacitor (or any single reactive component) across an input signal Vin, whilst the output signal Vout is taken from across the capacitor. The cut-off frequency or -3dB point, can be found using the formula, fc = 1/(2πRC). The phase angle of the output signal at fc and is -45^o for a Low Pass Filter.

The gain of the filter or any filter for that matter, is generally expressed in **Decibels** and is a function of the output value divided by its corresponding input value and is given as:

$$\textit{Gain in dB} = 20 \log \frac{Vout}{Vin}$$

Applications of passive Low Pass Filters are in audio amplifiers and speaker systems to direct the lower frequency bass signals to the larger bass speakers or to reduce any high frequency noise or "hiss" type distortion. When used like this in audio applications the low pass filter is sometimes called a "high-cut", or "treble cut" filter.

If we were to reverse the positions of the resistor and capacitor in the circuit so that the output voltage is now taken from across the resistor, we would have a circuit that produces an output frequency response curve similar to that of a *High Pass Filter*, and this is discussed in the next tutorial.

Time Constant

Until now we have been interested in the frequency response of a low pass filter and that the filters cut-off frequency (fc) is the product of the resistance (R) and the capacitance (C) in the circuit with respect to some specified frequency point and that by altering any one of the two components alters this cut-off frequency point by either increasing it or decreasing it.

We also know that the phase shift of the circuit lags behind that of the input signal due to the time required to charge and then discharge the capacitor as the sine wave changes. This combination of R and C produces a charging and discharging effect on the capacitor known as its **Time Constant** (τ) of the circuit as seen in the *RC Circuit* tutorials giving the filter a response in the time domain.

The time constant, **tau** (τ), is related to the cut-off frequency fc as.

$$\tau = RC = \frac{1}{2\pi fc}$$

or expressed in terms of the cut-off frequency, fc as.

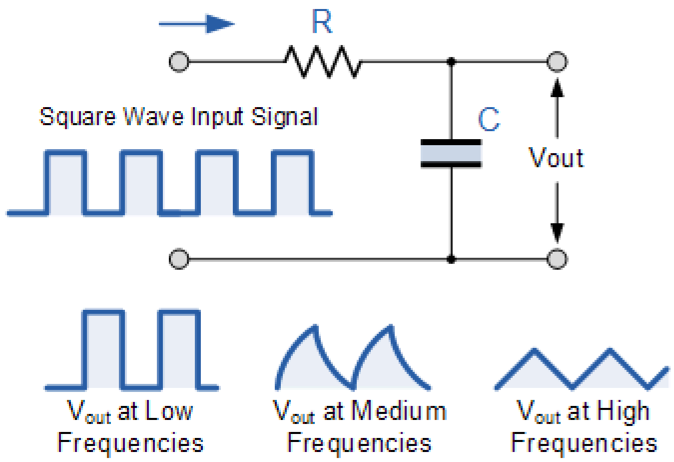
$$fc = \frac{1}{2\pi RC} \ or \ \frac{1}{2\pi\tau}$$

The output voltage, Vout depends upon the time constant and the frequency of the input signal. With a sinusoidal signal that changes smoothly over time, the circuit behaves as a simple 1st order low pass filter as we have seen above. But what if we were to change the input signal to that of a "square wave" shaped ON/OFF type signal that has an almost vertical step input, what would happen to our filter circuit now. The output response of the circuit would change dramatically and produce another type of circuit known commonly as an **Integrator**.

The RC Integrator

The **Integrator** is basically a low pass filter circuit operating in the time domain that converts a square wave "step" response input signal into a triangular shaped waveform output as the capacitor charges and discharges. A **Triangular** waveform consists of alternate but equal, positive and negative ramps. As seen below, if the RC time constant is long compared to the time period of the input waveform the resultant output waveform will be triangular in shape and the higher the input frequency the lower will be the output amplitude compared to that of the input.

The RC Integrator Circuit



This then makes this type of circuit ideal for converting one type of electronic signal to another for use in wave-generating or wave-shaping circuits.

http://www.electronics-tutorials.ws/filter/filter_1.html